Pushing Economies (and Students) Outside the Factor Price Equalization Zone

Paul Oslington and Isaac Towers

ABSTRACT: Despite overwhelming empirical evidence for the failure of factor price equalization most teaching of international trade theory (even at the graduate level) assumes that economies are incompletely specialized and factor price equalization holds. The behavior of trading economies in the absence of factor price equalization is not well understood, and some major textbook treatments err. The authors map regions of specialization and diversification for standard competitive economies and show how outputs, good and factor prices change as economies move within and across different regions of diversification and specialization. Two examples of how the analysis can enrich graduate-level trade teaching are given – the substitutability of goods trade and factor movements, and debates over the trade and inequality.

Keywords: integrated equilibrium, factor price equalization, teaching, trade
JEL Codes: A20, F11, F16

---

* Paul Oslington is Professor of Economics, Australian Catholic University
7 Mount Street North Sydney, NSW 2060, Australia.
Email: paul.oslington@acu.edu.au Telephone: 61 2 9739 2868
Isaac Towers is Lecturer in Mathematics at University of New South Wales/Australian Defence Force Academy. The authors thank seminar participants at University of New South Wales, Australian National University, and Princeton University, and the anonymous referees for helpful comments.

The teaching of international trade theory concentrates on fully diversified economies where factor price equalization holds. This is so at both undergraduate and graduate levels, and whether the topic is a workhorse competitive trade model or more exotic imperfectly competitive and dynamic models. The standard graduate textbook (Feenstra 2004, 22-25) discusses the empirical and policy importance of breakdowns of factor price equalization, but offers only brief comments on the complications involved in modelling trading economies in the absence of factor price equalization. The only textbooks to deal with the issue, Dixit and Norman (1980) and Bhagwati Srinivasan and Panagaria (1996) err in their analyses of nonfactor price equalization cases.

We know surprisingly little about the behavior of trading economies in the absence of factor price equalization, even for the simplest competitive models. Krugman (1995) in his survey commented that determining what happens outside the factor price equalization region is a "fairly nasty business"(p1247) and Deardorff (2001, 143) that we are “surprisingly ignorant”. The best model available, Leamer (1987) dealt with a three factor n-good m-country world economy with Leontieff technology. It is a brilliant paper but the lack of response of factor proportions when factor prices change outside the factor price equalization region is a limitation in relation to the issue considered here. Students also struggle with the simplex diagrams used in the paper.

We build on the widely used technique of integrated equilibrium analysis to map regions of specialization and diversification for trading worlds with different factor endowments. As in the existing treatments of Dixit and Norman (1980) and Bhagwati Srinivasan and Panagaria (1996), the focus is on a standard 2*2*2 trading world. Some simple diagrams of the behavior of goods and factor prices outside the factor price equalization zone are generated for a particular Cobb-Douglas technology. For teachers, pushing students outside the factor price equalization zone emphasizes how special are the standard textbook results, and enriches discussion of questions such as the substitutability of goods trade and factor movements, and the relationship between trade and inequality.

INTEGRATED EQUILIBRIUM ANALYSIS

The technique of integrated equilibrium analysis considers distributions of the world endowment of factors of production between the countries that support different patterns of specialization and diversification. The technique was developed by Dixit and Norman (1980, 100-125), who took up Samuelson's (1949, 194-195) parable of an angel splitting the world endowment of factors between countries. It allowed Dixit and Norman to cut through the previous debate on factor price equalization by changing the question from what regularity conditions on technology guaranteed factor price equalization with an assumed pattern of international specialization, to a question of what joint restrictions on technology, preferences and factor endowments supported factor price equalization.[1] Integrated equilibrium analysis has been extremely fruitful: Deardorff (1994) further clarified the conditions for factor price equalization; Helpman and Krugman (1985) and Kreickemeier and Nelson (2006) have extended it to trading worlds with imperfect competition; Davis (1998) called it "a truly global approach" when deriving startling results about the consequences for countries' factor markets of factor accumulation in different parts of the world.

The simplest and most widely used model with two countries, two factors and two goods will be our starting point. Assumptions are standard: perfect competition, concave constant returns to scale technology that is the same across the world, and identical homothetic preferences. It will be assumed that equilibrium factor proportions are unique, and that a positive quantity of each good is produced somewhere in the world.

The equilibrium for a world not divided into countries (or equivalently with free movement of goods and factors between countries) is shown in Figure 1, with equilibrium conditions in the appendix. The dimensions of the box are the world endowment of the factors, unskilled labor L and skilled labor (or human capital) K. Equilibrium factor usage vectors for the two products X and Y are shown. The slopes of the vectors indicate factor proportions and their lengths indicate outputs of the goods, and are drawn for a particular technology and preferences that will be held constant throughout the analysis. The steeper vector for good X indicates its technology is relatively unskilled labor intensive.

Now consider splitting the world endowment of the factors between countries A and B in the proportions represented by V in Figure 2. Because V is within the shaded parallelogram (the area enclosed by the factor usage vectors from Figure 1) both countries produce both goods using the same factor proportions as the undivided world. Factor prices and goods prices will be identical to the undivided world. Individuals in the countries will consume the products in the same proportions as the undivided world, as preferences are identical homothetic, so the factor content of consumption in the two countries will be a point on the diagonal of the box such as C. The net factor content of trade will thus be the vector VC. The factor content of trade in each good can be shown by drawing factor content rays through C that intersect the factor usage vectors for the two goods. The difference between the factor content of production and factor content of consumption of each good is the factor content of trade in each good, and these factor contents of trade (A exports X, B exports Y) are shown dotted in Figure 2. This is the factor price equalization case.

For splits of the endowment outside the shaded parallelogram in Figure 2 such replication of the integrated equilibrium is not possible and factor price equalization breaks down. This has been widely noted in the literature, but there is considerable uncertainty about what exactly happens. Dixit and Norman commented (1980, 113): "In order to be able to say what happens outside the factor price equalization region, we need more information concerning technology and demand functions" and that this can "make matters very complicated".

None of the discussions in the literature of what happens outside the factor price equalization region are completely accurate. Dixit and Norman's textbook, an excellent and widely used reference, errs in suggesting that there are four regions of specialization outside the factor price equalization region[2] (Dixit and Norman 1980, 113-114, especially Figure 4.4). As we will show below there are in fact six regions - they missed the possibility that both countries specialize completely in different goods. Bhagwati, Srinivasan and Panagaria (1996, 87-90) repeat the error that there are four regions and wrongly characterize the countries as producing both goods for those endowment distributions. There is no satisfactory account in the literature of what happens outside the factor price equalization region, and certainly none available to graduate students and their teachers.

WHAT HAPPENS OUTSIDE THE FACTOR PRICE EQUALIZATION REGION?

The correct regions of specialization and diversification are illustrated in Figure 3. The illustrated case has been generated using *Matlab* for Cobb-Douglas production and utility functions, with production share of K in X of 0.45, share of K in Y of 0.55, and consumption shares of 0.5 for the two products. Full equilibrium conditions for the different regions are given in the appendix.

The regions are best explained by considering how a trading world switches between equilibria as endowments change. Begin with an endowment split in the diversification and factor price equalization (FPE) region in Figure 3 where conditions (1)-(9) given in the appendix hold.[3] Change the distribution of the world factor endowment so country A has less of the world endowment of skilled labor and country B has more. Factor and goods prices do not change, but in country B the output of the labor intensive good X will fall and Y rise, following the Rybczynski Theorem.[4] Eventually the output of X in country B will fall to zero at the boundary of the diversification region. Any further increase in the endowment of skill in country B will make it impossible to fully employ its endowment of both factors producing both products at the integrated equilibrium factor proportions. There is not enough labor to absorb all country B's skill at these proportions, and to maintain full employment production of the labor-intensive good X must cease and Y must be produced with more skill intensive-techniques.

Now the economy is in the specialization region in Figure 3 where conditions (10)-(17) in the appendix hold. Consider taking further skill from country A and giving it to country B. The responses are now complicated by goods price, factor price and factor proportion changes that occur outside the factor price equalization region. Moving into the specialization region, in country A the output of the skill-intensive good Y will fall and the output of X rise, (the hypothetical Rybczynski response at constant goods prices) and the output of Y in country B will rise (country B is devoting all its resources to Y and now has more skill). The overall effect is a reduction in the relative world supply of Y at constant goods prices, which forces up the relative world price of the skill-intensive good Y. The return to skill in country A rises and the skill intensity of both goods falls. In B the skill intensity of Y must rise as it the only good produced, and return to skill falls. The goods price change also has an effect on consumers, reducing relative demand for good Y. There is also a feedback effect on production (i.e., moderating the hypothetical Rybczynski response above) from the goods price change. As we continue transfering skill from A to B the reinforcing production and demand effects will close down the Y industry in country A.

As the Y industry in A ceases production we enter the extreme specialization region in Figure 3. This is the region where conditions (18)-(24) in the appendix apply and each country produces a single good X in A and Y in B. It is the region previous authors have missed. For the Cobb-Douglas technology and preferences the extreme specialization region has the convex lens shape in Figure 3, although for other technologies this need not be the case [5] [6].

There are three analogous specialization regions and one extreme specialization regions as shown in Figure 3.

A question that has been discussed in the literature is the likelihood of factor price equalization. On the perhaps dubious assumption (see comments by Deardorff 1994, 174) that the endowment split is a random variable, uniformly distributed over the box (think of Samuelson's angel with eyes closed dropping a knife on a box representing the world economy) then the probability of factor price equalization is the area of the diversification region relative to the box. Deardorff (1994) showed that the more disparate are the factor intensities of the products the greater the area spanned by the integrated equilibrium factor usage vectors and the greater will be the likelihood of factor price equalization for given endowments. Empirical evidence (e.g., Debaere and Demiroglu 2003; Schott 2003) on factor proportions and endowments is not encouraging for factor price equalization.

GOODS AND FACTOR PRICES OUTSIDE THE FACTOR PRICE EQUALIZATION REGION

Comparative static responses to endowment changes within regions can be obtained by manipulating equilibrium conditions in the appendix (10)-(17) for the specialization region and (18)-(24) for the extreme specialization region. However, the global approach we are taking allows consideration of what happens across boundaries of regions. Figures 4-8 map goods prices p^Y and factor prices w^A , w^B, r^A , r^B for all possible endowment splits. The sharp changes and reversals of slope as we move across the boundaries of regions, especially moving from specialization to extreme specialization, show the limitations of restricting attention to comparative static responses within regions.

EXTENSIONS

Higher Dimensions

Integrated equilibrium analysis can be easily extended to situations with more goods than factors. The hexagonal factor price equalization region with three goods, and indeterminacy of the pattern of production and trade was described by Dixit and Norman (1980, 114-121) and Helpman and Krugman (1985, 15). The situation outside the factor price equalization region with three goods is less clear. It seems there will be regions of extreme specialization where each country produces one good, regions of specialization where countries produce a good in common, but we have not been able to derive expressions for boundaries.

Increasing the number of factors to three means we have a three-dimensional factor quantity box (as illustrated by Dixit and Norman 1980, 123) not spanned by the two factor usage vectors, so factor price equalization will be a fluke case.

Unemployment

Integrated equilibrium analysis can be generalized to situations of minimum-wage unemployment, provided the minimum-wage is common to both countries (otherwise the analysis becomes degenerate because specifying the proportion of the world endowment subject to the minimum wage fixes the endowment split). The factor price equalization region for economies with minimum wage unemployment is derived and compared to the full employment case in Oslington (2006, ch 7). Davis (1998) and Kreickemeier and Nelson (2006) provided good discussions of integrated equilibrium models with unemployment. Oslington (2006, ch 8) considered an equilibrium with unemployment outside the factor price equalization region.

Non-Traded Goods

Integrated equilibrium analysis has been extended to cover nontraded goods by Helpman and Krugman (1985, 19-22). Making a previously traded good nontraded reduces the size of the factor price equalization region, potentially altering pattern of production and prices.

VALUE FOR STUDENTS

The greatest benefit for students is in seeing how special are the diversified factor price equalization equilibria that dominate the textbooks. Seeing this encourages students to think more deeply about other fundamental assumptions – to question and generalize in a way that develops a research mentality

Exposing students to non-factor price equalization cases enriches discussion of a number of questions commonly discussed in international economics courses. Two examples are briefly discussed below.

Trade and Factor Movements

An important implication of the standard trade model is that goods trade and factor movements are perfect substitutes. In the world of the standard models there is no need to worry about barriers to international factor flows as the opening up of goods trade will achieve the same result of equalizing countries' factor prices. The result goes back to Mundell (1957), and Wong (1995) provide a comprehensive contemporary treatment.

Integrated equilibrium diagrams are an ideal tool for considering the implications of factor mobility, as they operates in factor quantity space. Figures 5-8 show prices of different factors in different countries in worlds with free movement of goods, but with factors tied to countries. The flat central regions of these figures indicate the endowment partitions where goods and factor trade are substitutes, and endowment partitions outside the central region where they are not. In these regions free migration in response to the illustrated factor price differentials would equalize factor prices in a way that trade cannot.

Trade and Wage Inequality

The contribution of increased international trade to widening gaps between skilled and unskilled wages in the United States has been a contentious issue in recent years. The argument made by Wood (1994), Borjas Freeman and Katz (1992) and others was that goods imported into the United States embody labor, that the bulk of these goods are unskilled labor intensive, and so trade will depress returns to unskilled labor in the United States. To labor economists (e.g., Freeman 2004) such an argument seemed to make sense – labor demand curves after all slope downwards. However trade economists (e.g., Leamer 2000) pointed out that in the standard Heckscher-Ohlin trade model factor prices are insensitive to endowment changes, so the argument is nonsense in such a framework. At best, trade economists suggested, the argument must be reformulated to operate through goods prices rather than factor quantities.

Integrated equilibrium analysis, as extended in this article, can help students sort through the issues. Leamer is exactly right for diversified two-factor-two-good economies – economies in the central zone of Figures 5-8. However, where economies are specialized there are the factor price effects depicted for the other regions of Figures 5-8.[7] A more detailed discussion of the impact of various changes in the world economy on inequality in a particular non-factor price equalization model with unemployment can be found in Oslington (2002, 2006).

CONCLUSION

A full mapping from endowments to patterns of production fills an important gap in trade theory, not least because the few existing discussions err. Our main purpose in this article is to explain the technique, provide some simple diagrams, and suggest how they can be used in teaching. We hope it will be useful to students and teachers of undergraduate as well as graduate courses in international economics.

NOTES

1. A common approach in the trade literature is to construct McKenzie (1955) cones of diversification, and argue that economies with endowment combinations inside the cones will be diversified, whereas those outside the cone will specialize. This is simple and sometimes useful, but will be misleading to the extent that goods prices change (as they will in a global economy when endowments or other parameters change), altering the position of the cones. This shortcoming was one of the reasons Dixit and Norman (1980) developed integrated equilibrium analysis.

2. In correspondence on this issue Avinash Dixit mentioned that his colleague Gene Grossman independently realized the error in the Dixit and Norman textbook (see Grossman 1990, and Grossman and Helpman 1991, 190). My letter to Avinash Dixit contained an error about the shape of one the regions and I thank him and Gene Grossman for pointing this out. Deardorff (1994, 169) included a diagram dividing the area outside the factor price equalization region into six regions, but drew linear boundaries that apply in the special case of fixed production coefficients. Courant and Deardorff (1992) considered the conceptually similar issue of lumpiness within countries, but with fixed goods prices.

3. For purposes of exposition assume the starting point is above (i.e., where A has more labor) the junction of the diversification and extreme specialization regions.

4. The property of the standard model that, at constant prices, and increase in the endowment of one factor increases the output of the industry that uses that factor relatively intensively and reduces the output of the other (or some other) industry (Deardorff 2006).

5. If the production technology had fixed coefficients then the boundaries of the extreme specialization region would be straight line extensions of the factor usage vectors that enclose the diversification region. However for other technologies where factor proportions are influenced by the factor price changes that occur outside the factor price equalization region, the closure of the Y industry in A that brings us into the extreme specialization region will be delayed

6. The boundary of the specialization and extreme specialization regions is the locus of L^A K^A obtained from solving (10)-(17) when setting Y^A=0 in (13) and (14). Needless to say it is an ugly expression even for the Cobb-Douglas case.

7. The other way of getting wage effects from trade is to vary the numbers of goods and factors, for instance the three-factor-two-good model mentioned in the extensions section, and explored by a number of trade economists (e.g., Leamer 1995).

References

Bhagwati, J., T. N. Srinivasan and A. Panagaria. 1996. *Lectures on international trade*. 2nd ed. Massachusetts: MIT Press.

Borjas, G. J., R. Freeman and L. Katz. 1992. On the labor market effects of immigration and trade, in *Immigration and the work force*, ed by G. Borjas and R. Freeman. Chicago: University of Chicago Press.

Courant, P. and A. Deardorff .1992. International trade with lumpy countries. *Journal of Political Economy* 100 (1): 198-210.

Davis, D. R. 1998. Does European unemployment prop up American wages? *American Economic Review* 88 (3): 478-94.

Deardorff, A. 1994. The possibility of factor price equalization revisited. *Journal of International Economics* 36 (1): 167-75.

Deardorff, A. 2001. Fragmentation in simple trade models. *North American Journal of Economics and Finance* 12 (2): 121-37.

Deardorff, A. 2006. Deardorff's glossary of international economics. http://www-personal.umich.edu/~alandear/glossary/r.html accessed April 22, 2008.

Deardorff, A. 2001. Fragmentation in simple trade models. *North American Journal of Economics and Finance* 12 (2): 121-37.

Debaere, P. and U. Demiroglu 2003. On the similarity of country endowments. *Journal of International Economics* 59 (1): 101-36.

Dixit, A. K. and V. Norman. 1980. *The theory of international trade - A dual general equilibrium approach.* Cambridge: Cambridge University Press.

Feenstra, R. C. 2004. *Advanced international trade: theory and evidence.* Princeton: Princeton University Press.

Freeman R. 2004. Trade wars: The exaggerated impact of trade in economic debate. *The World Economy* 27 (1): 1-23.

Grossman, G. M. 1990. *2x2x2: Two mistakes in the literature on the two-good-two-factor model of international trade*. Unpublished manuscript.

Grossman, G. M. and E. Helpman. 1991. *Innovation and growth in the global economy* Massachusetts: MIT Press.

Helpman, E. and P. Krugman. 1985. *Market structure and foreign trade.* Massachusetts: MIT Press.

Kreickemeier, U. and D. Nelson. 2006. Fair wages, unemployment and technological change in a global economy. *Journal of International Economics* 70 (2) 451-69.

Krugman, P. 1995. Increasing returns, imperfect competition and the positive theory of international trade, in *The handbook of international economics,* ed by G. Grossman and K. Rogoff. Amsterdam: North Holland.

Leamer, E.E. 1987. Paths of development in the three-Factor, N-good general equilibrium model. *Journal of Political Economy* 95 (5): 961-99.

Leamer, E.E. 1995. *The Heckscher-Ohlin model in theory and practice.* Princeton: New Jersey, Princeton Studies in International Finance No 77.

Leamer, E.E. 2000. What's the use of factor contents? *Journal of International Economics* 50 (1): 17-49.

McKenzie, L. 1955. Equality of factor prices in world trade. *Econometrica* 23 (3): 239-57.

Mundell, R. A. 1957. International trade and factor mobility. *American Economic Review* 4: 321-35.

Oslington, P. 2002. Factor market linkages in a global economy. *Economics Letters* 76 (1): 85-93.

Oslington, P. 2006. *The theory of international trade and unemployment.* Aldershot: Edward Elgar.

Samuelson, P. A. 1949. International factor price equalization once again. *Economic Journal* 59 (June): 181-97.

Schott, P. K. 2003. One size fits all? Heckscher-Ohlin specialization in global production. *American Economic Review* 93 (3): 686-708.

Wong, K.Y. 1995. *International trade in goods and factor mobility.* Massachusetts: MIT Press.

Wood, A. 1994. *North-south trade, employment and inequality.* Oxford: Clarendon Press.

Woodland, A. D. 1982. *International trade and resource allocation.* Amsterdam: North Holland.

Appendix – Equilibrium Conditions

The general form of the equilibrium conditions is given below. The case illustrated in Figures 3-8 has Cobb-Douglas production and utility functions, with production share of K in X of 0.45, share of K in Y of 0.55, and consumption shares of 0.5 for the two products. These equilibrium conditions are expressed in terms of minimum unit cost functions c(w,r), whose derivatives with respect to the factor prices $c_w(w,r)$ and $c_r(w,r)$ are input-output coefficients - see Woodland (1982).

For the factor price equalization region, equilibrium conditions are:

Zero profit for each product produced in each country (X is the numeraire).

$$(1)\quad 1 = c^X(r^A, w^A)$$

$$(2)\quad 1 = c^X(r^B, w^B)$$

$$(3)\quad p^Y = c^Y(r^A, w^A)$$

$$(4)\quad p^Y = c^Y(r^B, w^B)$$

Full employment of each factor in each country ($\overline{L}$ and $\overline{K}$ are world endowments)

$$(5)\quad c_w^X(r^A, w^A)\, X^A + c_w^Y(r^A, w^A)\, Y^A = L^A$$

$$(6)\quad c_r^X(r^A, w^A)\, X^A + c_r^Y(r^A, w^A)\, Y^A = K^A$$

$$(7)\quad c_w^X(r^B, w^B)\, X^B + c_w^Y(r^B, w^B)\, Y^B = \overline{L} - L^A$$

$$(8)\quad c_r^X(r^B, w^B)\, X^B + c_r^Y(r^B, w^B)\, Y^B = \overline{K} - K^A$$

Demand

$$(9)\quad \frac{X^A + X^B}{p^Y Y^A + p^Y Y^B} = \frac{1-\sigma^Y}{\sigma^Y}$$

These conditions yield equilibrium values of p^Y, w^A, w^B, r^A, r^B, X^A, Y^A, X^B and Y^B.

In the specialization marked in figure 3 where product X is not produced in B, conditions are:

Zero profit for each product produced in each country:

$$(10)\quad 1 = c^X(r^A, w^A)$$

$$(11)\quad p^Y = c^Y(r^A, w^A)$$

$$(12)\quad p^Y = c^Y(r^B, w^B)$$

Full employment of each factor in each country:

$$(13)\quad c_w^X(r^A, w^A)\, X^A + c_w^Y(r^A, w^A)\, Y^A = L^A$$

$$(14)\quad c_r^X(r^A, w^A)\, X^A + c_r^Y(r^A, w^A)\, Y^A = K^A$$

$$(15)\quad c_w^Y(r^B, w^B)\, Y^B = \overline{L} - L^A$$

$$(16)\quad c_r^Y(r^B, w^B)\, Y^B = \overline{K} - K^A$$

Demand

$$(17)\quad \frac{X^A}{p^Y Y^A + p^Y Y^B} = \frac{1-\sigma^Y}{\sigma^Y}$$

These conditions yield equilibrium values of p^Y, w^A, w^B, r^A, r^B, X^A, Y^A and Y^B.

Equilibrium conditions in the extreme specialization region are:

Zero profit for each product produced in each country:

$$(18)\quad 1 = c^X(r^A, w^A)$$

$$(19)\quad p^Y = c^Y(r^B, w^B)$$

Full employment of each factor in each country:

$$(20)\quad c_w^X(r^A, w^A)\, X^A = L^A$$

$$(21)\quad c_r^X(r^A, w^A)\, X^A = K^A$$

$$(22)\quad c_w^Y(r^B, w^B)\, Y^B = \overline{L} - L^A$$

$$(23)\quad c_r^Y(r^B, w^B)\, Y^B = \overline{K} - K^A$$

Demand

$$(24)\quad \frac{X^A}{p^Y Y^B} = \frac{1-\sigma^Y}{\sigma^Y}$$

These conditions yield equilibrium values of p^Y, w^A,w^B, r^A,r^B, X^A and Y^B.

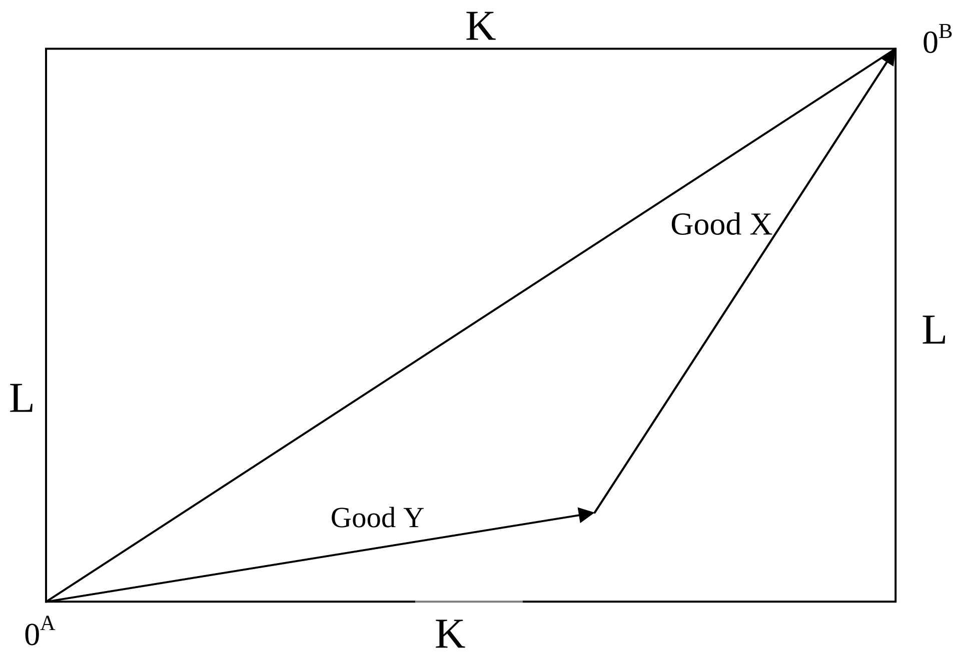


FIGURE 1. Undivided world.

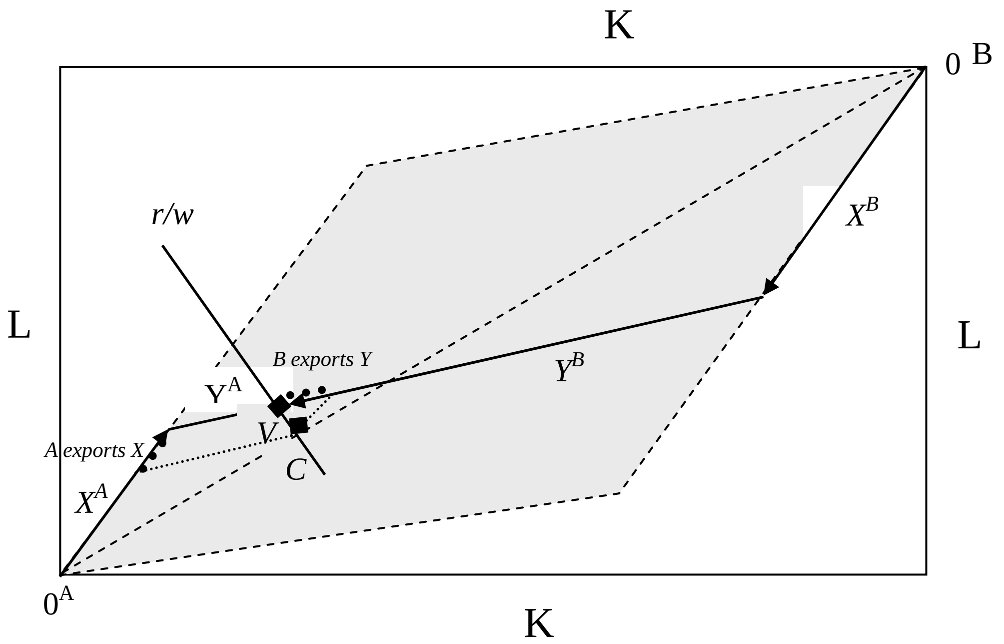
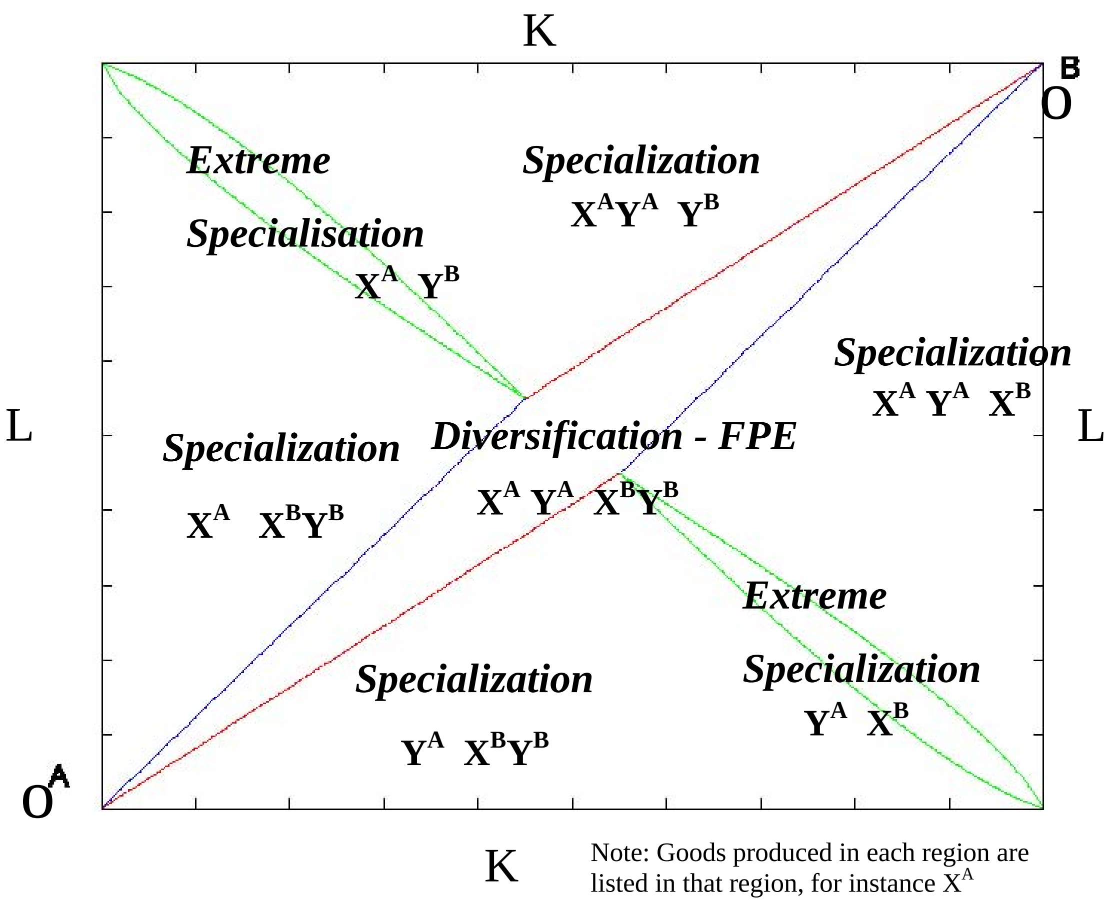
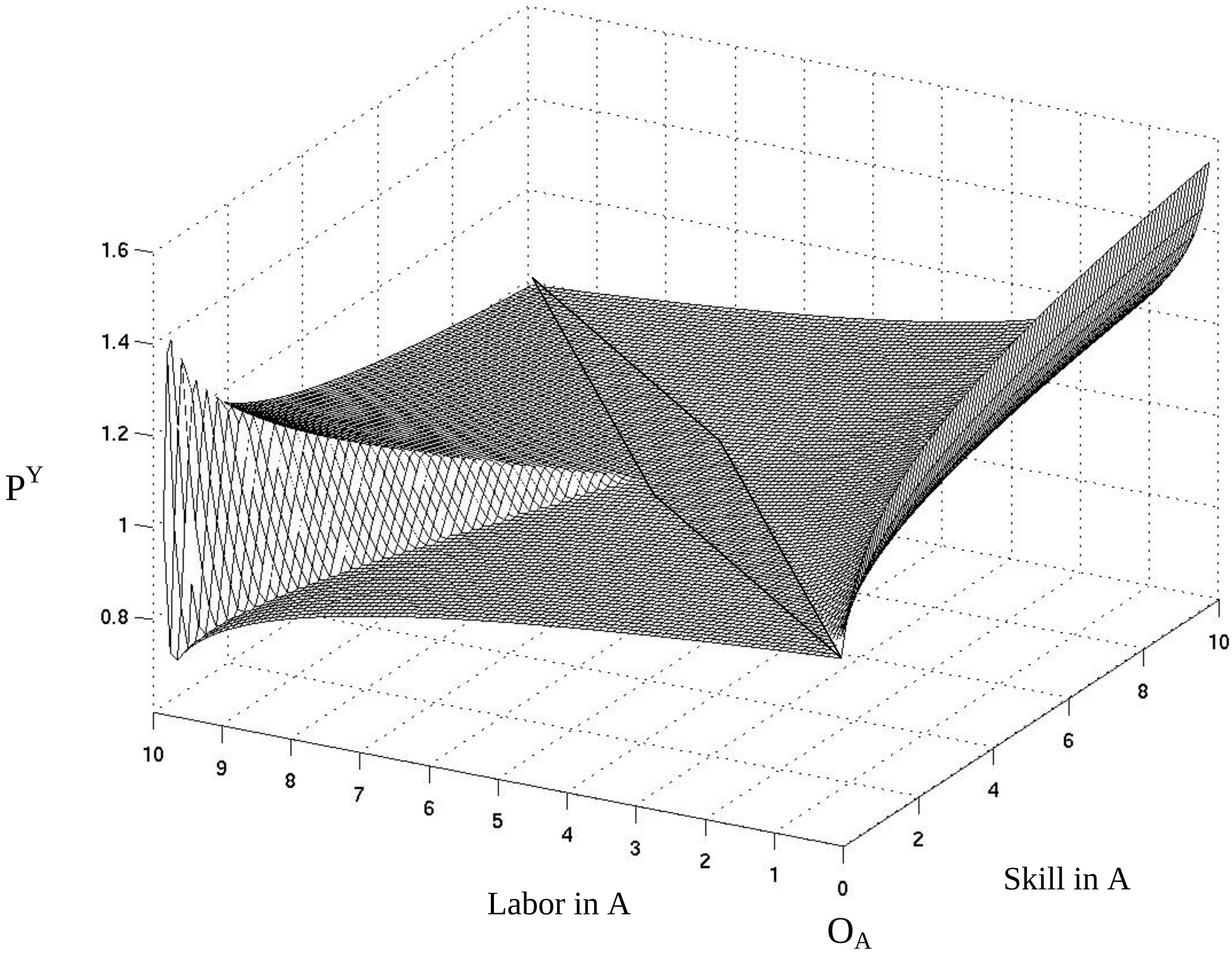


FIGURE 2. Integrated Equilibrium.



Note: Goods produced in each region are listed in that region, for instance X^A means good X is produced in country A .

FIGURE 3. Regions of specialization and diversification with Cobb-Douglas technology (X is L intensive).



Note: Figure 4 shows the p^Y surface from a perspective that gives the clearest view of the transitions from the specialization to extreme specialization regions in the left portion of the figure, but at the cost of obscuring the symmetric right portion. All the surfaces in Figures 4-8 are symmetric.

FIGURE 4. Price of product Y.

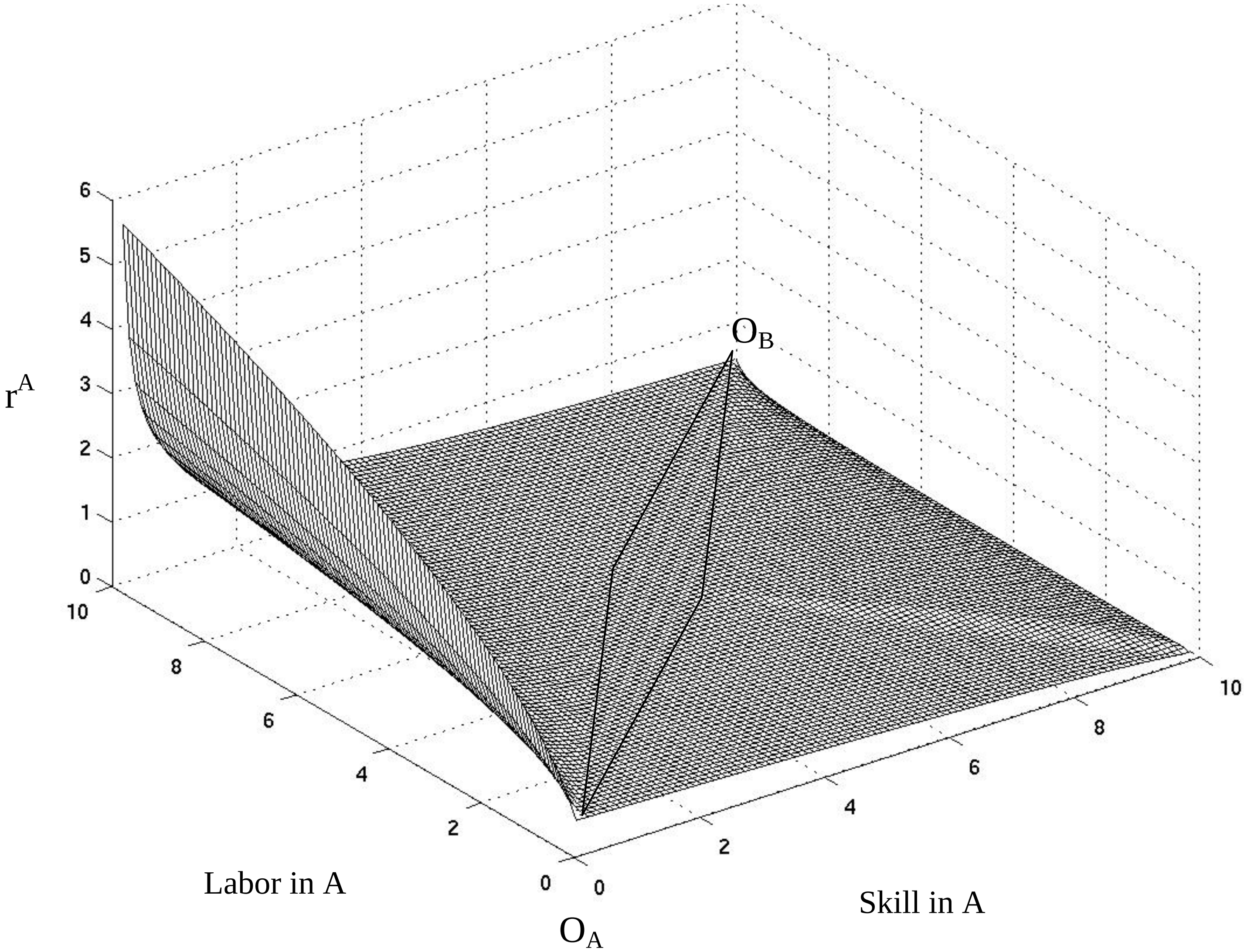


FIGURE 5. Skill returns in country A.

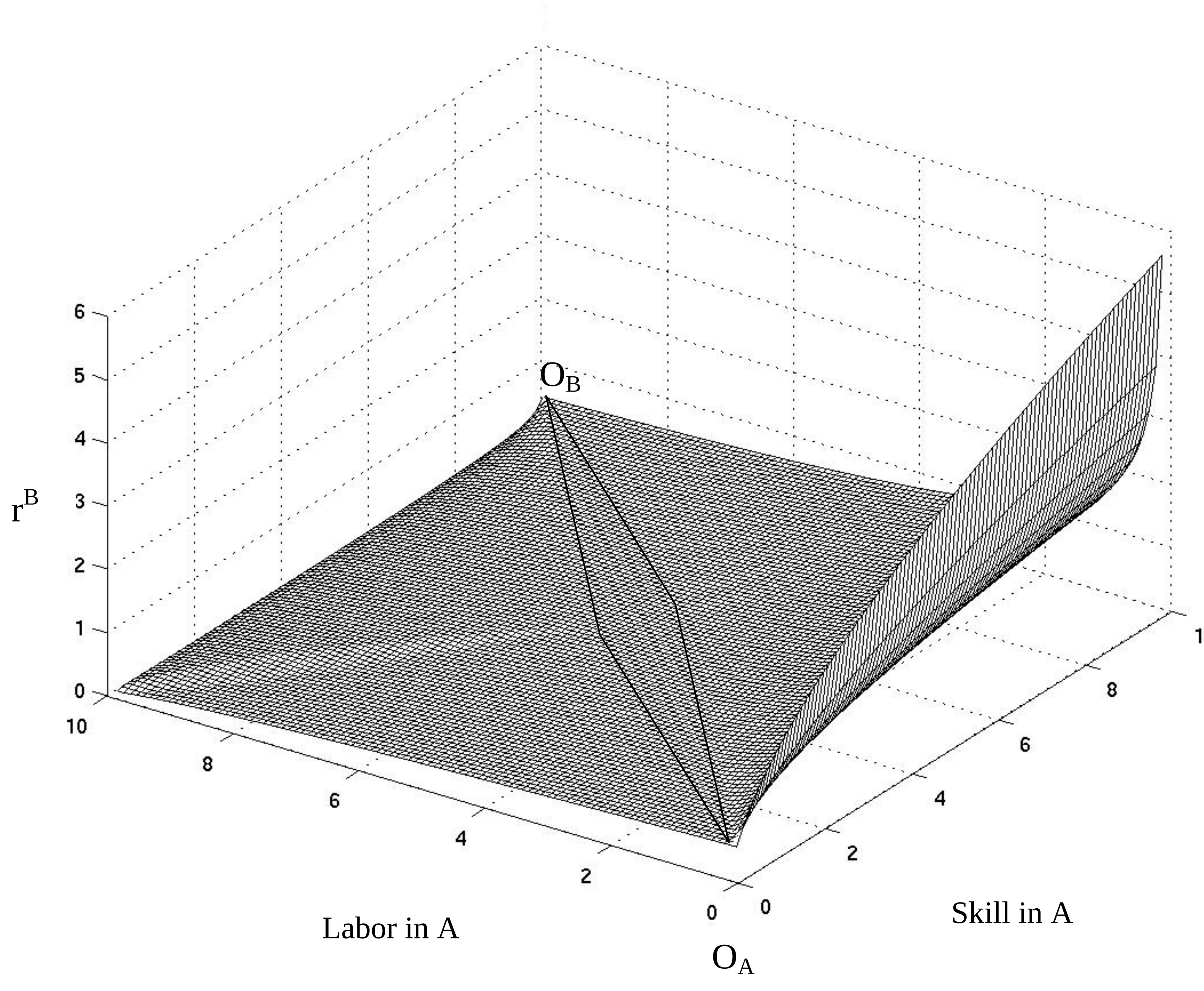


FIGURE 6. Skill Returns in Country B.

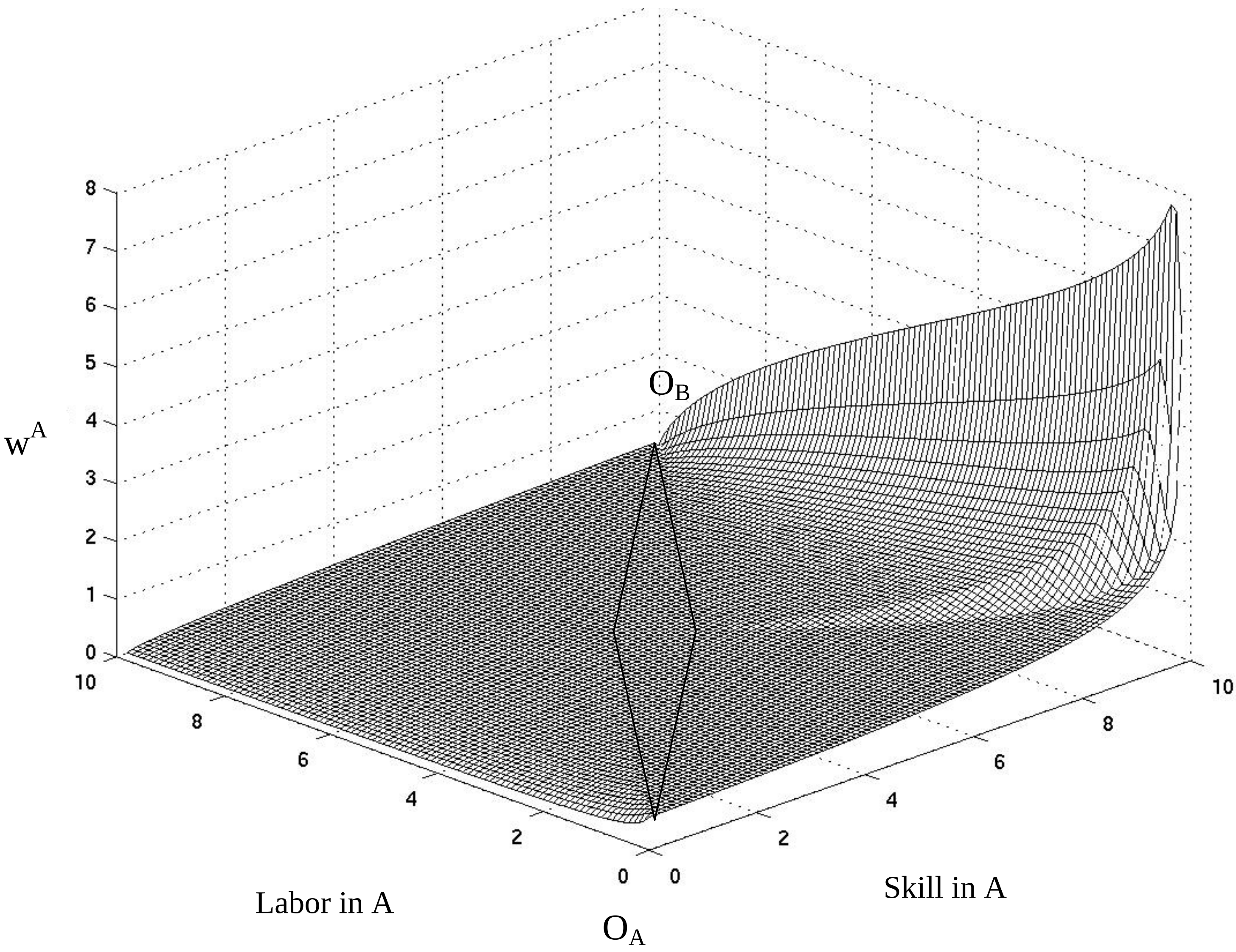


FIGURE 7. Wages in Country A.

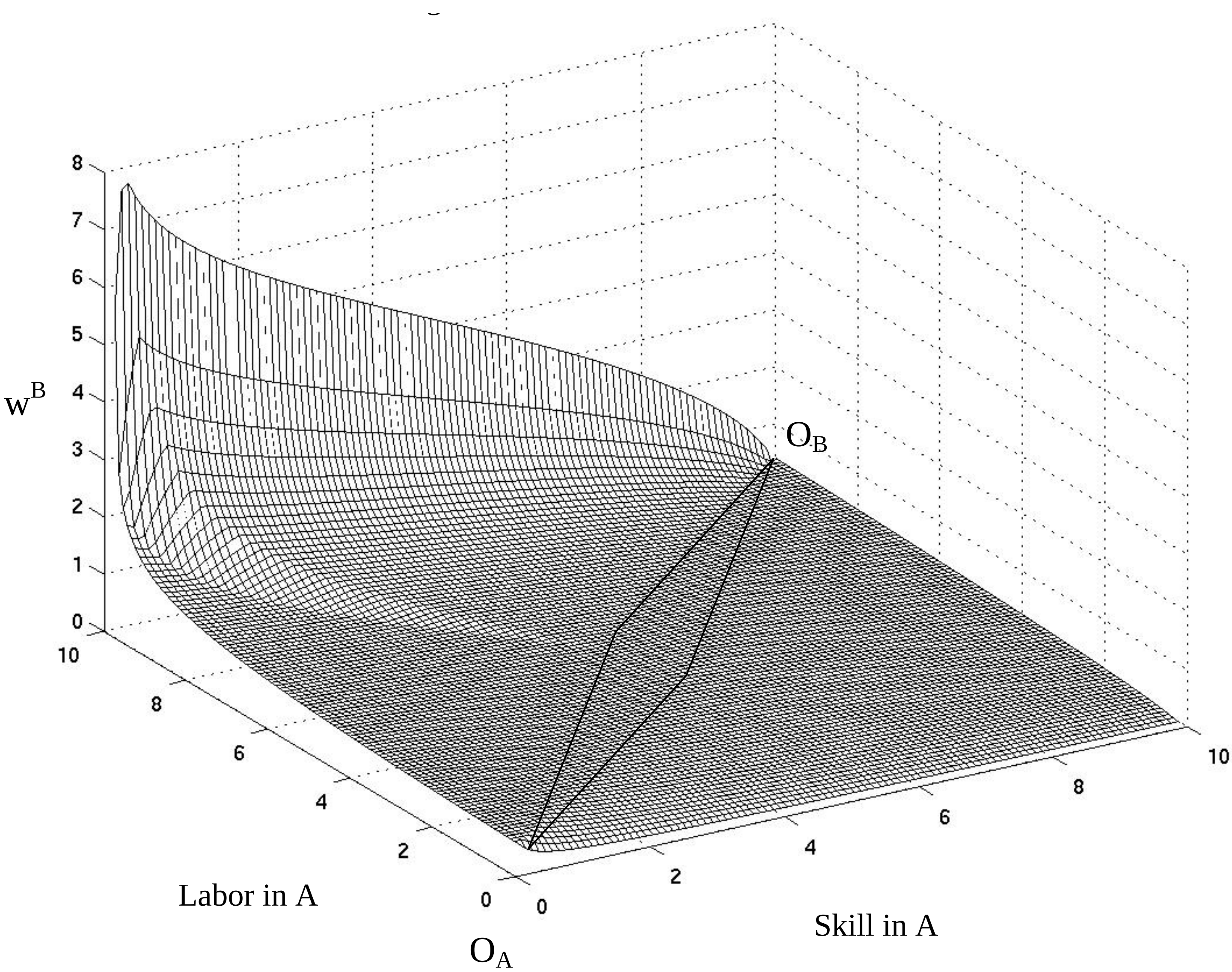


FIGURE 8. Wages in Country B.